Comparison between Induction Motor and Transformer

An induction motor is considered as a transformer with a rotating short-circuited secondary. The stator winding corresponds to transformer primary and the rotor winding corresponds to transformer secondary. However, there are differences:

(i) The induction motor has an air gap, but the transformer has a core. Therefore, the magnetizing current in an induction motor is much larger than that of the transformer. For example, in induction motor is about 30-50 % of rated current whereas it is only 1-5% of rated current in a transformer.

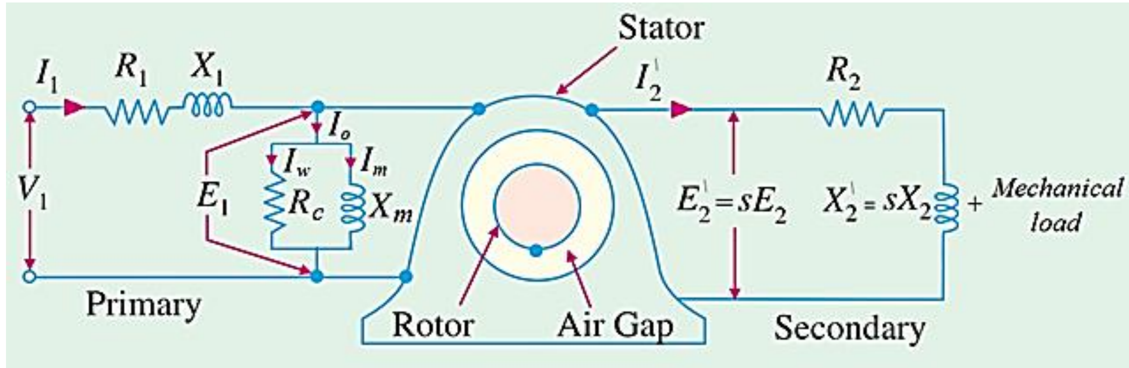
(ii) In an induction motor, the stator and rotor windings are distributed around the periphery of the air gap rather than concentrated on a core as in a transformer (*i.e.*, in the motor: leakage flux passes easily through the air gap) (*i.e.*, in transformer: small amount of leakage flux passes <u>away from</u> the core). Therefore, the leakage reactances of stator and rotor windings quite large compared to that of a transformer.

(iii) In an induction motor, the input is electrical but output is mechanical. However, in a transformer, input as well as output are electrical.

(iv) Unlike the transformer, in the induction motor the rotor voltage, reactance and frequency are proportional to the slip *s*. At any slip *s*, rotor values:

e.m.f/phase ($E_2^{\backslash}=sE_2$) … reactance/phase ($X_2^{\backslash}=sX_2$) … frequency ($f^{\backslash}=sf$)

Equivalent Circuit of 3-Phase Induction Motor at any Slip



- In a 3-phase IM, the stator winding is connected to 3-phase supply and the rotor winding is short-circuited. The energy is transferred *magnetically* from the stator winding to the short-circuited rotor winding. Therefore, an induction motor may be considered as a transformer with a rotating secondary (short-circuited).
- Then the stator winding corresponds to transformer primary and the rotor windings corresponds to transformer secondary. So, the equivalent circuit of an induction motor will be similar to that of a transformer. Figure shows the equivalent circuit *per phase* for an induction motor.
- The applied voltage V_1 produces a magnetic flux which links the stator winding (i.e., primary) as well as the rotor winding (i.e., secondary). Thus; self-induced e.m.f. E_1 is induced in the stator winding and mutually induced e.m.f. $E_2^{\backslash}$ is induced in the rotor winding (where: $E_2^{\backslash} = sE_2 = s\,kE_1$ … k is a transformation ratio).
- **Why we draw the branch of magnetization as shown in the above figure…see MagBranch.MP4**

Stator circuit of the Equivalent Circuit

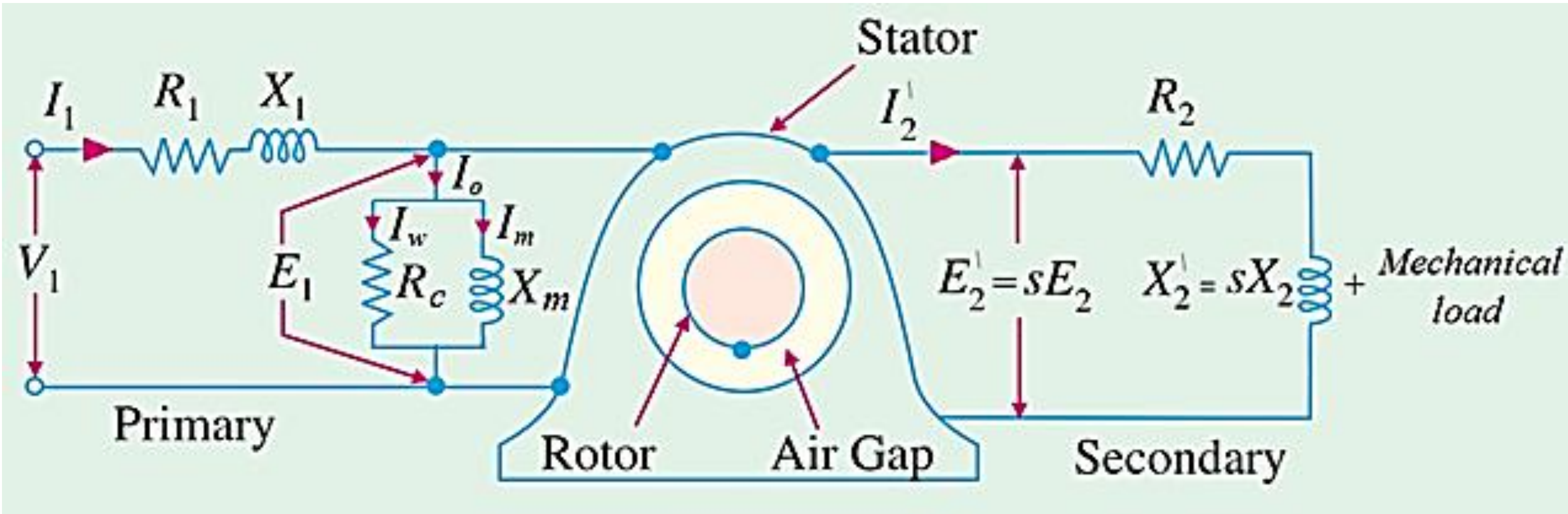


Fig. 1: equivalent circuit of induction motor per phase at any slip

V_1 : supply voltage or stator terminal voltage/phase.
R_1 : stator resistance/phase.
X_1 : stator leakage reactance/phase.
I_1 : supply current or stator current/phase.
E_1 : induced emf voltage/phase in the stator windings (by self-induction).

$$\overline{V}_1 = \overline{E}_1 + \overline{I}_1(R_1 + jX_1)$$

Stator circuit of the Equivalent Circuit …

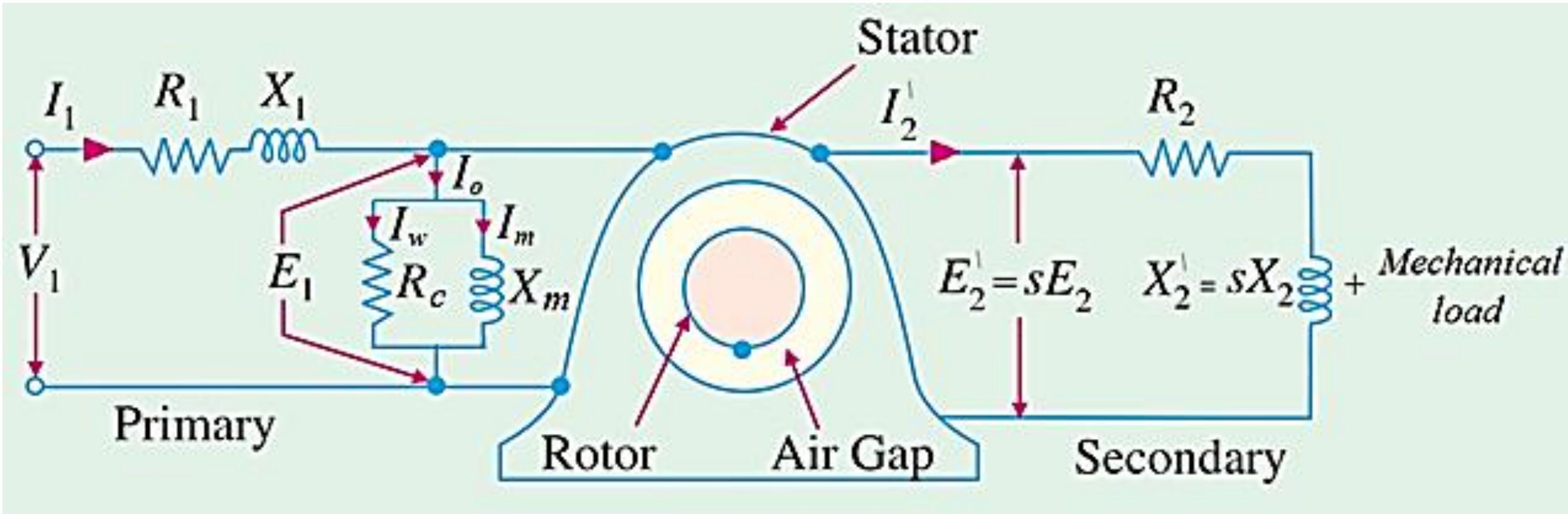


Fig. 1: equivalent circuit of induction motor per phase at any slip

I_o : no-load current/phase (passes at when the motor has no-load).

I_w : working or iron loss current component/phase (which causes the no-load motor losses).

I_m : magnetization current component/phase (which setup the magnetic flux in the core and the airgap).

$\overline{I_o} = \overline{I_w} + \overline{I_m}$ Also, $\overline{I_o} = I_o \cos\phi_o - jI_o \sin\phi_o$ ϕ_o *angle between* E_1 & I_o

Stator circuit of the Equivalent Circuit …

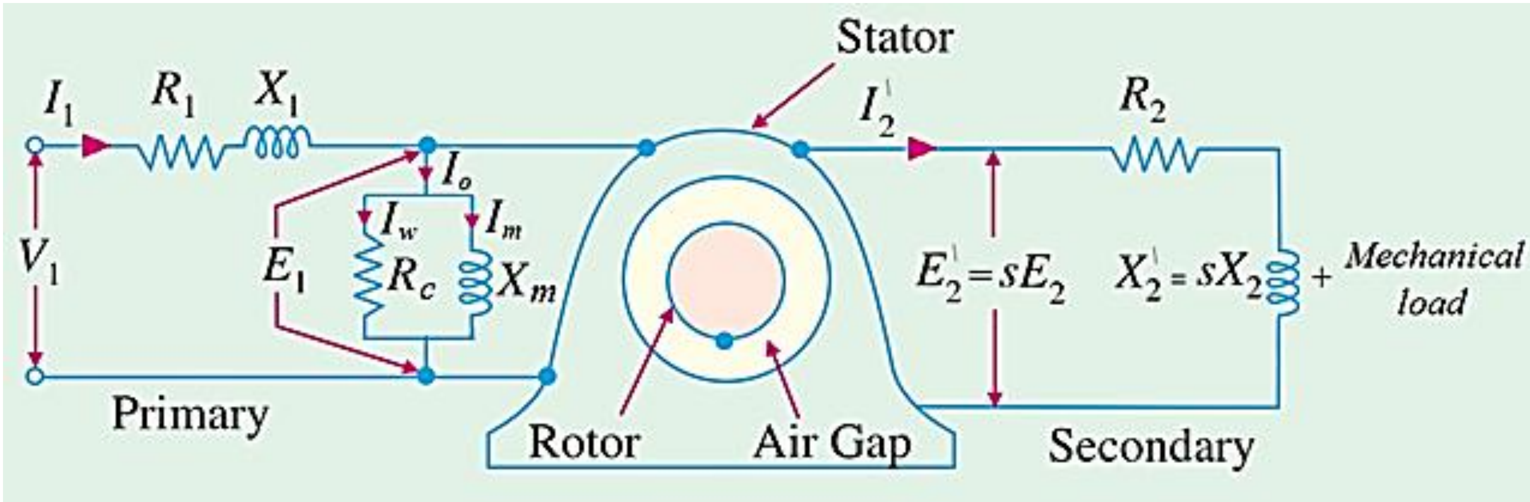


Fig. 1: equivalent circuit of induction motor per phase at any slip

- From the magnetization branch:

$$\overline{E_1} = \overline{I_o}.(R_c \,//\, jX_m) = \overline{I_o}.Z_{mag.}$$

$$\overline{E_1} = \overline{I_w}.R_c \qquad \overline{E_1} = \overline{I_m}.jX_m$$

Rotor circuit of the Equivalent Circuit

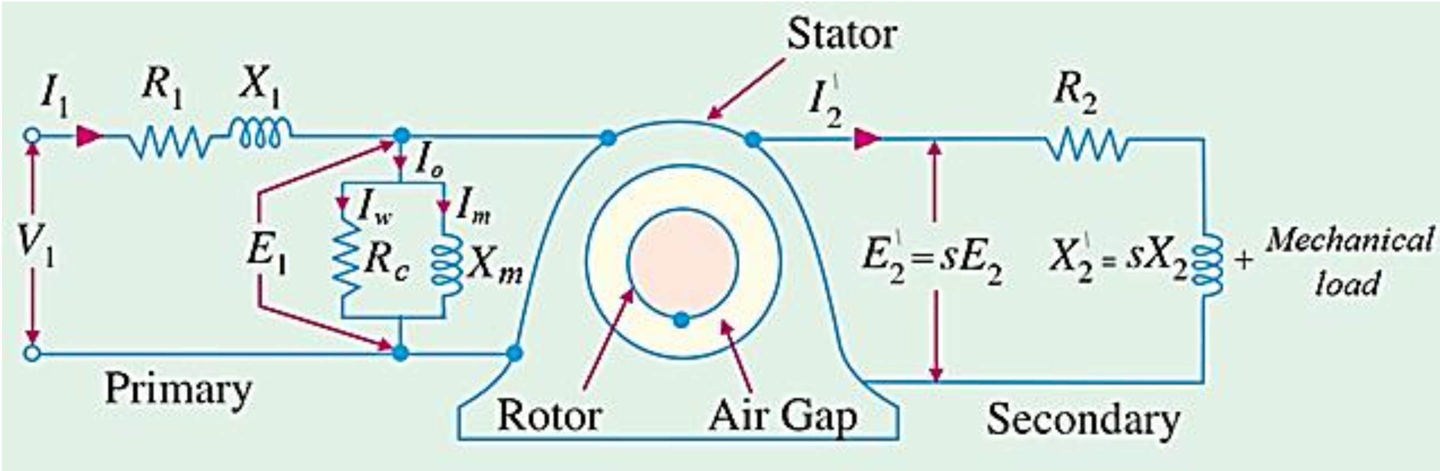


Fig. 1: equivalent circuit of induction motor per phase at any slip

- Since the rotor winding is short-circuited, the whole of induced e.m.f/phase $E_2^\backslash$ is used to setup a circulating rotor current $I_2^\backslash$.

$E_2^\backslash$: induced emf voltage/phase in the rotor windings (by mutual-induction).
$I_2^\backslash$: rotor current/phase.
R_2 : rotor resistance/phase.
$X_2^\backslash$: rotor leakage reactance/phase.

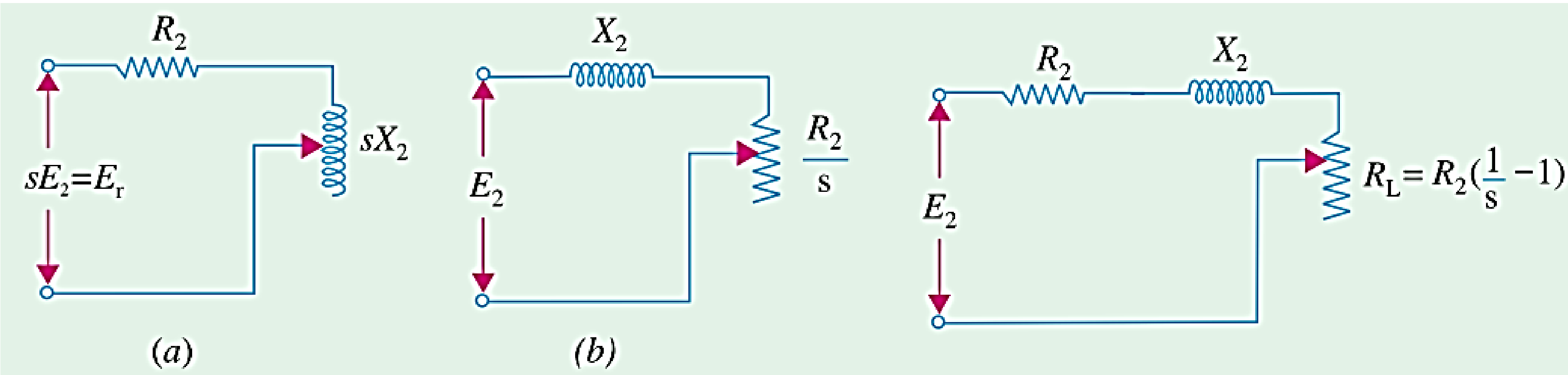
$$\overline{E_2^\backslash} = \overline{I_2^\backslash}(R_2 + jX_2^\backslash) \quad i.e.: \quad I_2^\backslash = \frac{E_2^\backslash}{\sqrt{R_2^2 + X_2^{\backslash 2}}} = \frac{E_2^\backslash}{\sqrt{R_2^2 + (sX_2)^2}}$$

Equivalent Circuit of the Rotor

- We shall now see how mechanical load of the motor is replaced by the equivalent electrical load. The rotor phase current is given by;

$$I_2^{\backslash} = \frac{E_2^{\backslash}}{\sqrt{R_2^2 + X_2^{\backslash 2}}} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \cdot \frac{\frac{1}{s}}{\frac{1}{s}} = \frac{E_2}{\sqrt{(\frac{R_2}{s})^2 + (X_2)^2}} = \frac{E_2}{\sqrt{(R_2 + R_2(\frac{1}{s} - 1))^2 + (X_2)^2}}$$

- From above equation: We now have a rotor circuit that has a fixed reactance X_2 connected in series with a variable resistance R_2/s and supplied with constant voltage E_2. The rotor circuit can be represented by the following figure. (all figures here are the same)



Equivalent Circuit of the Rotor …

$$\because \frac{R_2}{s} = R_2 + R_2(\tfrac{1}{s} - 1) = R_2 + R_L$$

- The rotor resistance in above equation has two components:

1- The first part R_2 is the rotor resistance itself and represents the rotor Cu losses.

2- The second part $R_2(\frac{1}{s}-1)$ is the load resistance R_L is the electrical equivalent of the mechanical load on the motor. In other words, the mechanical load on the induction motor can be electrically represented by $R_L = R_2(\frac{1}{s}-1)$.

Transformer Equivalent Circuit of Induction Motor

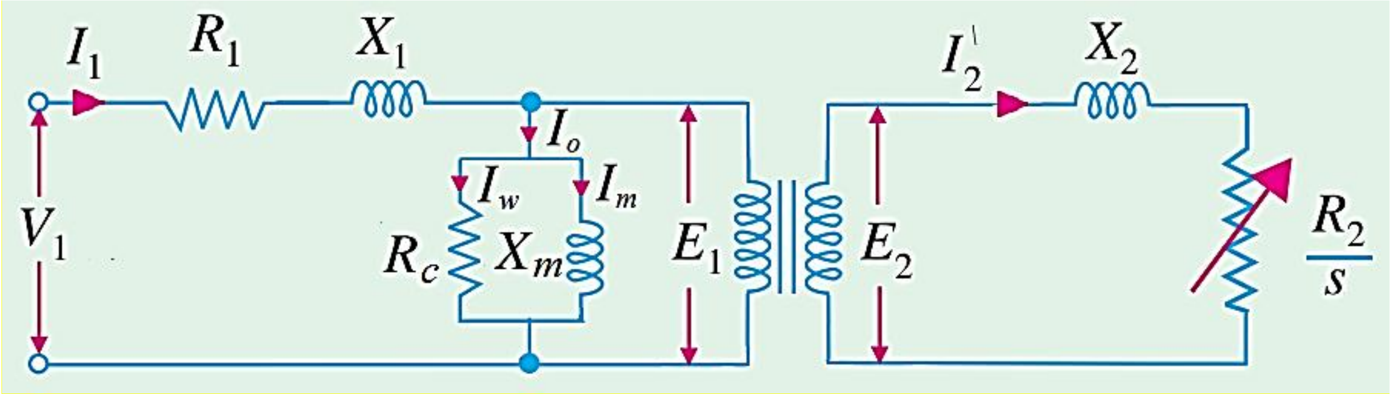


Fig. 2: eqt. circuit of induction motor per phase at any slip

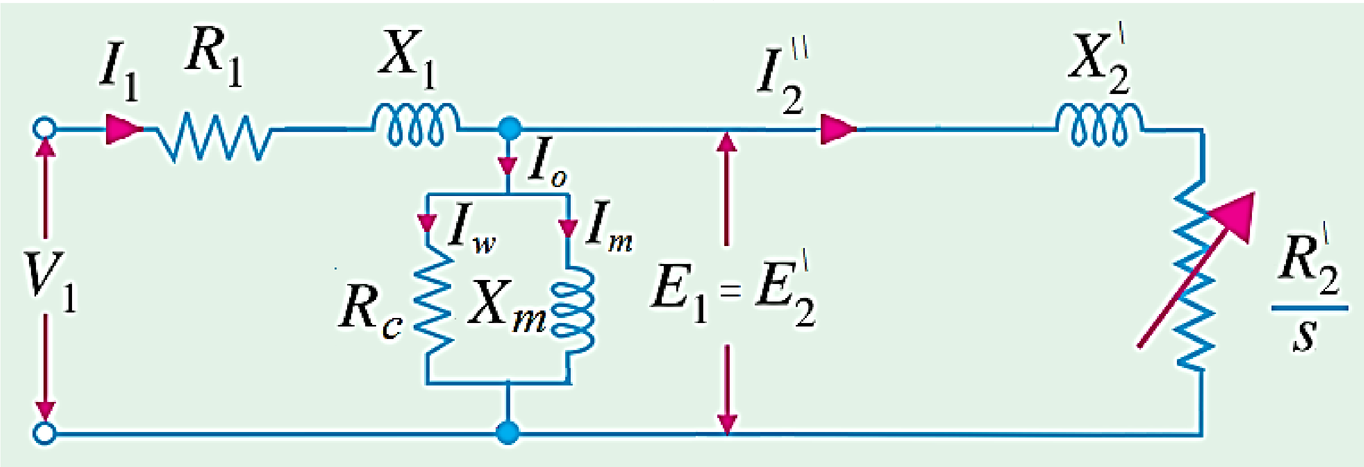


Fig. 2: eqt. circuit of induction motor per phase at any slip <u>referred to stator side</u>

Equivalent Circuit of Induction Motor Referred to Stator Side

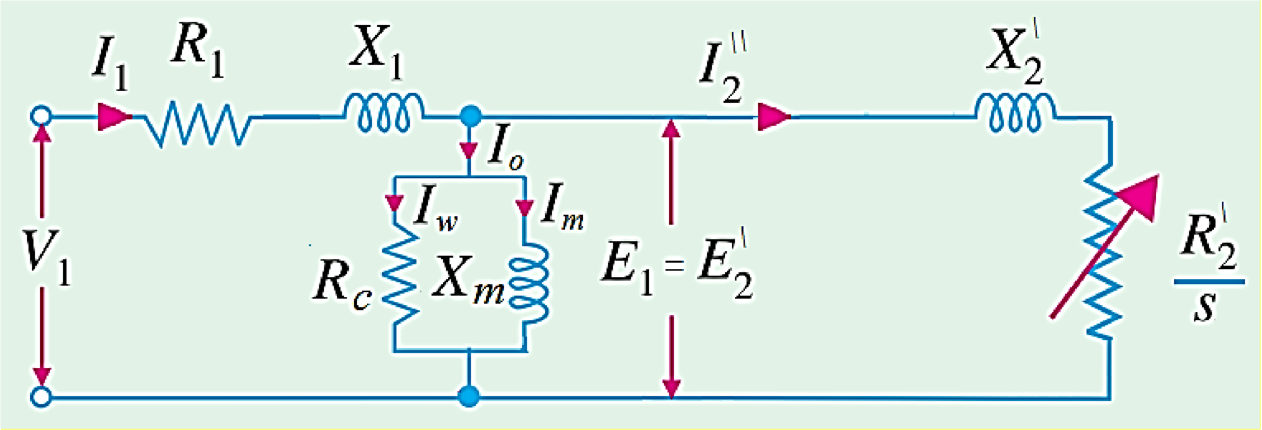


Fig. 2: eqt. circuit of induction motor per phase at any slip referred to stator side

- Referring to stator side is made based on the transformation ratio *k*. $k = \frac{E_2}{E_1}$

$E_1 = E_2^{\backslash} = \frac{E_2}{k}$ rotor induced emf/phase referred to stator side

$I_2^{\backslash\backslash} = kI_2^{\backslash}$ rotor current/phase referred to stator side

$X_2^{\backslash} = \frac{X_2}{k^2}$ rotor leakage reactance/phase referred to stator side

$\frac{R_2^{\backslash}}{s} = \frac{(\frac{R_2}{k^2})}{s}$ (rotor resistance/phase + load resistance/phase) referred to stator side

Equivalent Circuit of Induction Motor Referred to Stator Side …

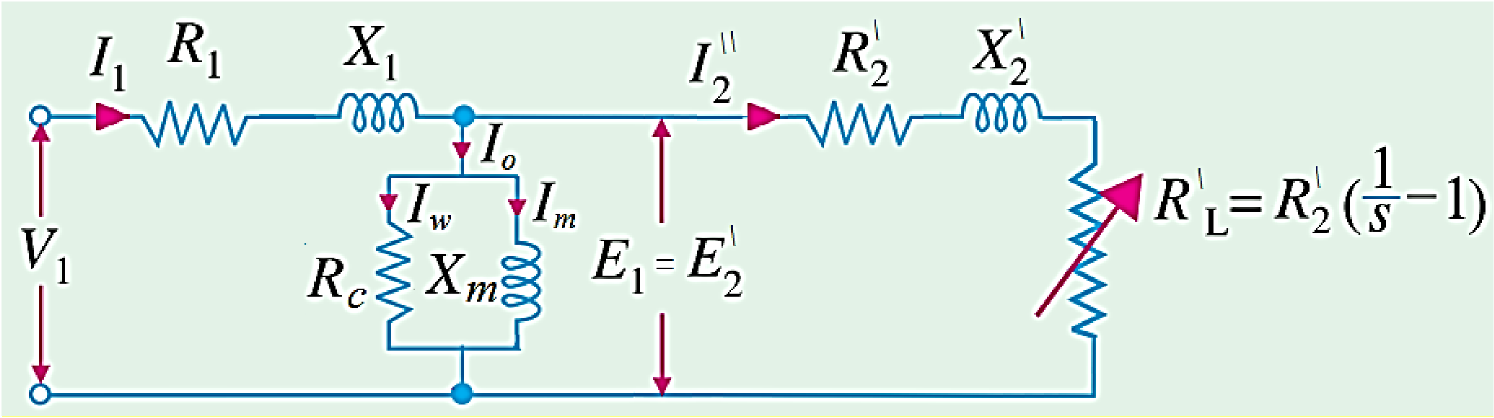


Fig. 3: eqt. circuit of induction motor per phase at any slip referred to stator side
(Fig. 3 is the same as Fig. 2)

- Earlier we say that:

$E_2^{\backslash}$ rotor induced emf/phase at any slip s. ……………..…. ($E_2^{\backslash} = s\, E_2$)
$X_2^{\backslash}$ rotor leakage reactance/phase at any slip s. ………… ($X_2^{\backslash} = s\, E_2$)
$I_2^{\backslash}$ rotor current/phase at any slip s. …………………….. ($I_2^{\backslash} = E_2^{\backslash}/Z_2^{\backslash}$)

- In studying the equivalent circuit (from Fig. 3):

$E_2^{\backslash}$ rotor induced emf/phase at any slip s referred to stator side. … ($E_2^{\backslash} = E_2/k$)
$X_2^{\backslash}$ rotor reactance/phase at any slip s referred to stator side. ……. ($X_2^{\backslash} = X_2/k^2$)
$I_2^{\backslash}$ rotor current/phase at any slip s referred to stator side. ……..… ($I_2^{\backslash} = E_2^{\backslash}/Z_2^{\backslash}$)
From Fig. 3 ……. $Z_2^{\backslash} = (R_2^{\backslash} + R_L^{\backslash}) + jX_2^{\backslash}$

Equivalent Circuit of Induction Motor Referred to Stator Side …

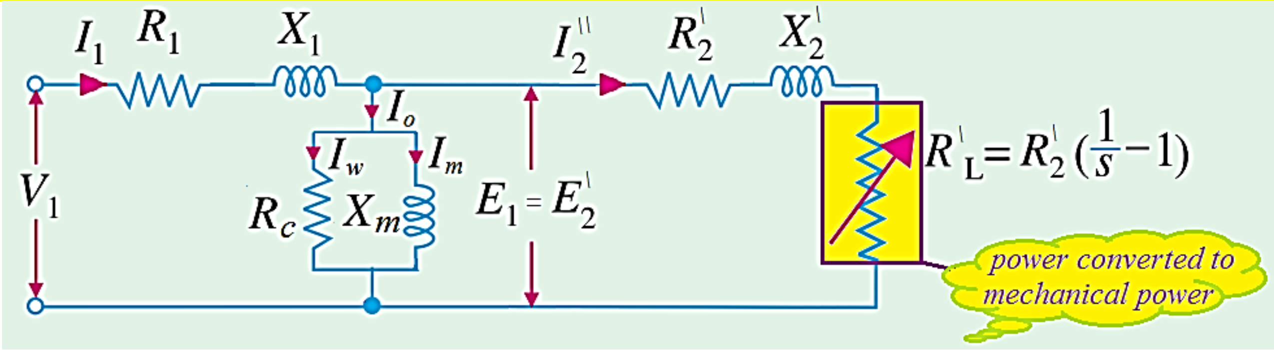


Fig. 3: eqt. circuit of induction motor per phase at any slip referred to stator side
(Fig. 3 is the same as Fig. 2)

Note that: the element $R_L^{\prime}$ is the equivalent electrical resistance related to the mechanical load on the motor. The following points may be noted from the equivalent circuit of the induction motor:

- At no-load (where $N_s \approx N$), the slip is practically zero and the load $R_L^{\prime}$ is infinite. This condition looks like a transformer whose secondary winding is open-circuited.
- At standstill (N=0), the slip is unity and the load $R_L^{\prime}$ is zero. This condition looks like a transformer whose secondary winding is short-circuited.
- When the motor is running under load, the value of $R_L^{\prime}$ will depend upon the value of the slip s. This condition resembles that in a transformer whose secondary is supplying variable and purely resistive load.
- $R_L^{\prime}$ depends on slip s. If the slip s increases, the load $R_L^{\prime}$ decreases and the rotor current increases and motor will develop more mechanical power. This is expected because the slip of the motor increases with the increase of load.

Power Relations from *Equivalent Circuit* of Induction Motor

(i) Stator input power : $P_{in} = 3V_1 I_1 \cos\phi_1$

(ii) Stator Cu losses : $P_{cu(stator)} = 3I_1^2 R_1$

(iii) Rotor input power : $P_{rotor} = \dfrac{3(I_2^{\backslash\backslash})^2 R_2^{\backslash}}{s}$

(iv) Rotor Cu losses : $P_{cu(rotor)} = 3(I_2^{\backslash\backslash})^2 R_2^{\backslash}$ or $P_{cu(rotor)} = s\,P_{rotor}$

(v) Total mechanical power developed by rotor : $P_{mech} = 3(I_2^{\backslash\backslash})^2 R_2^{\backslash}(\dfrac{1}{s} - 1)$

Or $$P_{mech} = P_{rotor} - P_{cu(rotor)} = \frac{3(I_2^{\backslash\backslash})^2 R_2^{\backslash}}{s} - 3(I_2^{\backslash\backslash})^2 R_2^{\backslash} = 3(I_2^{\backslash\backslash})^2 R_2^{\backslash}(\frac{1}{s} - 1)$$

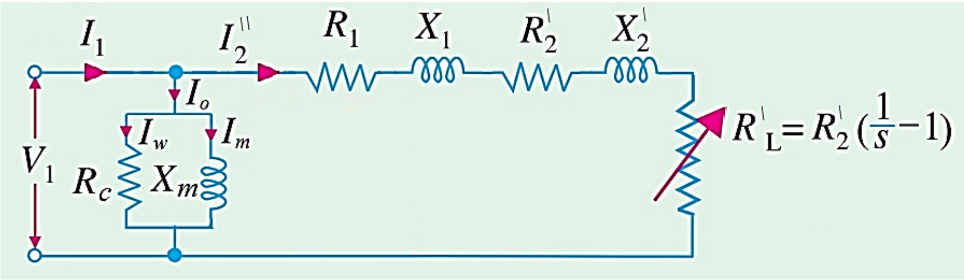
(vi) Total or gross torque developed by rotor :

$$T_{gross} = \frac{P_{rotor}}{\omega_s} = \frac{3(I_2^{\backslash\backslash})^2 \frac{R_2^{\backslash}}{s}}{\frac{2\pi N_s}{60}} = 9.55\,\frac{3(I_2^{\backslash\backslash})^2 \frac{R_2^{\backslash}}{s}}{N_s} = 9.55\,\frac{P_{rotor}}{N_s}$$

N_s is the synchronous speed in *rpm*

Note: the shaft torque T_{sh} less than T_{gross} by friction and windage losses.

Approximate Equivalent Circuit of Induction Motor



As in case of a transformer, the approximate equivalent circuit of an induction motor is obtained by shifting the shunt branch ($R_c \backslash\backslash jX_m$) to the input terminals as shown in Figure (Fig.: *approximate eqt. circuit of IM per phase referred to stator side*). This step has been taken on the assumption that voltage drop in R_1 and X_1 is small and the terminal voltage V_1 does not appreciably differ from the induced voltage E_1. From the above approximate circuit, we note that:

(i) Unlike a power transformer, the magnetic circuit of induction motor has an air-gap. Therefore, the exciting current of induction motor (30 to 40% of full-load current) is much higher than that of the transformer. Consequently, the exact equivalent circuit must be used for accurate results.

(ii) The values of X_1 and $X_2^\backslash$ in an induction motor are larger than the corresponding ones to be found in the transformer. This fact does not justify the use of approximate equivalent circuit.

(iii) In a transformer, the windings are concentrated whereas in an induction motor, the windings are distributed. This affects the transformation ratio.

(iv) Despite the above drawbacks of approximate equivalent circuit, this approximate circuit is satisfactory for large motors but not preferred for small motors.

Examples on Equivalent Circuit of Induction Motor

Example 34.55. *A 3-phase, star-connected 400 V, 50-Hz, 4-pole induction motor has the following per phase parameters in ohms, referred to the stators.*

$$R_1 = 0.15,\ X_1 = 0.45,\ R_2' = 0.12,\ X_2' = 0.45,\ X_m = 28.5$$

Compute the stator current and power factor when the motor is operated at rated voltage and frequency with s = 0.04. **(Elect. Machines, A.M.I.E. Sec. B, 1990)**

[Ans: $\overline{I_1} = 71.058\angle -21.47A$, $pf = \cos\phi_1 = \cos 21.47 = 0.93 lag$]

Example 34.59. *A 115V, 60-Hz, 3-phase, Y-Connected, 6-pole induction motor has an equivalent T-circuit consisting of stator impedance of (0.07 + j 0.3) Ω and an equivalent rotor impedance at standstill of (0.08 + j 0.3) Ω. Magnetising branch has G_o = 0.022 mho, B_o = 0.158 mho. Find **(a)** secondary current **(b)** primary current **(c)** primary p.f. **(d)** gross power output **(e)** gross torque **(f)** input **(g)** gross efficiency by using approximate equivalent circuit. Assume a slip of 2% .*

[Ans: (a) $16.15\angle -8.4A$ (b) $21.66\angle -36.38A$ (c) $0.805\, lag$ (d) $P_{gross} = 3I_2^{\backslash\backslash 2} R_L^{\backslash} = 3067 Watt$
(e) $24.9 Nm$ (f) $P_{in} = 3473 Watt$ (g) $\%\eta = \frac{P_{gross}}{P_{in}} \times 100 = 88.3\%$]